Published by Institute of Physics Publishing for SISSA/ISAS
Received: *December 6, 2004*
Revised: *December 16, 2004*
Accepted: *December 21, 2004*

# Dbrane near NS5-branes: with electromagnetic field

**Bin Chen,**[a] **Miao Li**[ab] **and Bo Sun**[bc]

[a] *Interdisciplinary Center of Theoretical Studies, Chinese Academy of Science*
*P.O. Box 2735, Beijing 100080, P.R. China*
[b] *Institute of Theoretical Physics, Chinese Academy of Sciences*
*P.O. Box 2735, Beijing 100080, P.R. China*
[c] *Graduate School of the Chinese Academy of Science*
*Beijing 100080, P.R. China*
*E-mail:* bchen@itp.ac.cn, mli@itp.ac.cn, sunbo@itp.ac.cn

Abstract: We study the D$p$-brane dynamics near NS5-branes with constant electromagnetic field. In the framework of effective Dirac-Born-Infeld action, we investigate the effect of the electromagnetic field on the D$p$-brane dynamics. The radial motion of the D$p$-brane on the transverse directions of NS5-branes can be mapped to a rolling tachyon in a constant NS $B_{\mu\nu}$ background. In the near throat region, the classical motion can be identified with the rescaled hairpin. After constructing the boundary state of the rescaled hairpin, we discuss the closed string emission of the D$p$-brane and find that the energy of the closed string emission is always finite in the presence of constant electric field. Taking the winding strings into account, the emitted energy is divergent, indicating that the emitted winding strings carry away most of the energy.

Keywords: D-branes, Tachyon Condensation.

## Contents



## 1. Introduction

The subject of time-dependent solutions in string theory is important since its study has been expected to provide us with the knowledge about the resolution of the black hole singularity and the cosmological singularity. The rolling tachyon, being an exactly solvable time-dependent model in the classical open string theory, has drawn much attention since proposed by A. Sen [1, 2]. The condensation of the rolling tachyon leads to closed string radiation and the relics of the closed string emission may be identified with the tachyon matter [3]–[5], which is a pressureless fluid. Nevertheless, the closed string emission remains an intriguing issue. Another remarkable fact is that the low energy effective action actually describes many aspects of the rolling tachyon condensation quite well [6]. The origin of this agreement has been investigated in [7].

In the study of the rolling tachyon, a very interesting generalization is to implement a constant NS $B_{\mu\nu}$ background, or a constant electromagnetic flux, on the worldvolume of the unstable D$p$-brane. In the case of the rolling tachyon with constant flux, the DBI action is still a good description of the dynamics of the rolling tachyon [17, 18]. It turns out that in the end of the tachyon condensation there is not only pressureless tachyon matter but also string fluid induced by the electric field [13]. It was also found that the presence of electric field can slow down the emission of closed strings. In particular, it has been shown that the emitted energy of closed strings without winding is finite and negligible [14]. However, after taking into account the winding strings, the emitted energy is divergent when $p \leq 3$, suggesting that most of the emitted energy goes to the highly wound closed strings [15].

Recently, D. Kutasov noticed that the behavior of D$p$-brane near NS5-branes is similar to the rolling tachyon [8]. Let the world-volume of NS5-branes lie along $x^0, \dots x^5$ and the one of D$p$-brane lie along $x^0, \dots x^p$ with $p \leq 5$. In both type II string theories, such a configuration breaks supersymmetry completely and is unstable. It was argued that the motion of D$p$-brane can be described reliably in a wide range by the DBI action in the supergravity background of NS5-branes. The radial motion of the D$p$-brane in the transverse direction $x^6, \ldots x^9$ can be mapped to the motion of the rolling tachyon after field redefinition. When the D$p$-brane enters the deep throat region of NS5-branes, it behaves like a supersymmetric hair-pin brane, which can be described by a boundary state of corresponding conformal field theory in the throat. The conformal field theory turns out to be $\mathbb{R}^{1,5} \times \mathrm{SU}(2)_k \times R_\phi$, where $\mathrm{SU}(2)_k$ is the three-sphere with radius $\sqrt{k}$ and $R_\phi$ is the CFT of a linear dilaton [16]. The boundary state of the hair-pin brane was constructed in [10]. The closed string emission of the D$p$-brane near NS5-branes was discussed in [11, 10]. Other related discussions can be found in [12].

In this paper, we will study the D$p$-brane dynamics near NS5-branes with an electromagnetic field along the worldvolume of D$p$-brane. Here we will use the effective DBI description to study various aspects of D$p$-brane dynamics. Using T-duality approach, we manage to construct the boundary state of the modified hairpin brane and study the closed string emission with and without the winding strings.

In section 2, we discuss the motion of D$p$-brane using the DBI action. In section 3, we will construct the boundary state of the modified hairpin brane. And in section 4 and 5, we discuss the closed string emission without and with the winding strings respectively. We conclude the paper in section 6.

## 2. DBI analysis

In this section we use the effective action (DBI action) for the D$p$-brane to analyze its dynamics near NS5-branes. The tension of a NS5-brane scales as $1/g_s^2$ while the tension of a $Dp$-brane scales as $1/g_s$, so it is natural to take NS5-branes' supergravity solution as a background for the D$p$-brane when the string coupling is weak.

The coordinates on the world-volume of $k$ coincident NS5-branes are $x^\mu$, $\mu = 0, 1, \ldots, 5$, and we use $x^n, n = 6, 7, 8, 9$ to label the four transverse dimensions. Let the D$p$-brane be parallel to NS5-branes and let the worldvolume of D$p$-brane lie along $x^0, \ldots x^p$ with $2 \leq p \leq 5$. Such a system breaks supersymmetry completely and is unstable.

Setting $r^2 = \sum_{n=6}^{9} x^n x^n$, the low energy supergravity solution of NS5-branes is

$$\begin{aligned} ds^2 &= dx^\mu dx_\mu + H(r) dx^n dx_n \equiv g_{MN} dx^M dx^N \\ \frac{g_s(\Phi)}{g_s} &= \exp(\Phi - \Phi_0) = \sqrt{H(r)} \\ H_{mnp} &= -\epsilon_{qmnp} \partial^q \Phi \,, \\ H(r) &= 1 + \frac{k l_s^2}{r^2} \,, \end{aligned} \tag{2.1}$$

where $H_{mnp}$ is the NS 2-form field strength, $g_s$ is the asymptotic string coupling, and $l_s$ is the string length unit.

Since there is an SO(4) rotational symmetry for the four transverse dimensions, without loss of generality we focus on the radial motion of a D$p$-brane. The dynamics of D$p$-brane is well described by the DBI action

$$S_p = -\tau_p \int d^{p+1}x e^{-(\Phi-\Phi_0)} \sqrt{-det(G_{\mu\nu} + B_{\mu\nu})} \tag{2.2}$$

where $\tau_p$ is the asymptotic tension of the D$p$-brane

$$\tau_p \sim \frac{1}{g_s l_s^{p+1}} \tag{2.3}$$

Now we turn on the electromagnetic field $B_{01} = e, B_{10} = -e, B_{12} = b, B_{21} = -b$. After pull back to the worldvolume of the Dbrane, it reads[1]

$$\mathbf{G}_{\mu\nu} = \begin{pmatrix} -1 + H\dot{r}^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{B}_{\mu\nu} = \begin{pmatrix} 0 & e & 0 & 0 \\ -e & 0 & b & 0 \\ 0 & -b & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.4}$$

The radial position of the Dbrane gives rise to a scalar field $r(x^\mu)$ on the worldvolume. We are only interested in the case when $r$ is just a function of $t$, so the action is

$$\begin{aligned} S_p &= -\tau_p V \int dt \frac{1}{\sqrt{H(r)}} \sqrt{(1+b^2)(1-H\dot{r}^2) - e^2} \\ &= -\tau_p V \int dt \sqrt{\frac{1}{H(r)}(1+b^2-e^2) - \dot{r}^2(1+b^2)}\,. \end{aligned} \tag{2.5}$$

Similar to the case without the electromagnetic flux, by defining a "tachyon" field $\frac{dT}{dr} = \sqrt{H(r)}$ one can map the above action to the DBI action of the rolling tachyon in the presence of constant electromagnetic field but with tachyon potential $V(T) = \frac{\tau_p}{\sqrt{H(r)}}$. Since we are mainly interested in the near throat region where CHS theory[16] works, a rescaled parameter is more convenient: take $R = g_s^{-1} r$ to be finite while $g_s \to 0, r \to 0$. In this limit $H(r)$ is simply $H(r) \sim r^{-2}$, thus the action can be written as:

$$\begin{aligned} S_p &= -\tau_p V \int dt \sqrt{\frac{g_s^2(1+b^2-e^2)}{kl_s^2} R^2 - g_s^2(1+b^2)\dot{R}^2} \\ &= -T_p V \int dt \frac{R}{\sqrt{k} l_s} \sqrt{(1+b^2-e^2) - (\frac{d}{dt}\log R)^2 (1+b^2) k l_s^2}\,, \end{aligned} \tag{2.6}$$

where $T_p \equiv \tau_p g_s$. Recall (2.3), the effective tension $T_p$ does not vanish in this limit.

In terms of the rescaled radial coordinate, the solution reads:

$$R = \frac{\sqrt{k} l_s}{A \cosh \frac{\sqrt{1+b^2-e^2} t}{l_s \sqrt{k(1+b^2)}}}\,, \tag{2.7}$$

[1] Here we take $p = 3$, it is straightforward to generalize to other cases.



JHEP12(2004)057

where A is an integration constant depending on the initial condition and is related to the conserved energy tensor $\rho$ as in (2.11), $A = T_p(1+b^2)/\rho\sqrt{1+b^2-e^2}$. The linear dilaton $\Phi = -\phi/\sqrt{k}l_s$, where $\phi$ is the proper distance, and

$$e^{-\frac{\phi}{\sqrt{k}l_s}} = A\cosh\frac{\sqrt{1+b^2-e^2}t}{l_s\sqrt{k(1+b^2)}}\,. \tag{2.8}$$

In late time, $t\to\infty$, one has

$$\phi \sim -\gamma^{-1}|t|\,, \tag{2.9}$$

with

$$\gamma^{-1} = \frac{\sqrt{1+b^2-e^2}}{\sqrt{(1+b^2)}} < 1\,. \tag{2.10}$$

So the Dbrane approaches a limiting speed $\gamma^{-1}$ in the late time. It is due to the presence of the electromagnetic field that the limiting speed is less than the speed of light.

It is remarkable that (2.8) is a modification of hairpin brane discussed in [10, 11]. We will show in the next section, by using T-duality approach, that one can construct the boundary state for this brane and then discuss the closed string emission issue.

It is easy to calculate the energy-momentum tensor. From (2.2), one has:

$$\begin{aligned}\delta S &= -\frac{\tau_p}{2}e^{-(\Phi-\Phi_0)}\sqrt{-\det(\mathbf{G}+\mathbf{B})}(\mathbf{G}+\mathbf{B})^{\mu\nu}(\delta g_{\mu\nu}+\delta b_{\mu\nu})\\ &= -\frac{\tau_p}{2}\frac{1}{\sqrt{H(r)}}\sqrt{-\det(\mathbf{G}+\mathbf{B})}(\mathbf{G}+\mathbf{B})^{\mu\nu}(\delta g_{\mu\nu}+\delta b_{\mu\nu})\end{aligned}$$

We get the energy-momentum tensor $T^{\mu\nu}$ and NS source $S^{\mu\nu}$. For the above solution (2.8), the non-vanishing components are:

$$\begin{aligned}
T^{00} &= \frac{T_p(1+b^2)}{A\sqrt{1+b^2-e^2}} = \rho\\
T^{11} &= \frac{\rho}{1+b^2}\left(\frac{(1+b^2-e^2)\tanh^2(\frac{\sqrt{1+b^2-e^2}\,t}{l_s\sqrt{k(1+b^2)}})}{1+b^2} - 1\right)\\
T^{22} &= \frac{\rho}{1+b^2}\left(\frac{(1+b^2-e^2)\tanh^2(\frac{\sqrt{1+b^2-e^2}\,t}{l_s\sqrt{k(1+b^2)}})}{1+b^2} - 1 + e^2\right)\\
T^{02} &= \frac{eb\rho}{1+b^2}\\
T^{33} &= \frac{\rho}{1+b^2}(1+b^2-e^2)\left[\tanh^2\left(\frac{\sqrt{1+b^2-e^2}\,t}{l_s\sqrt{k(1+b^2)}}\right) - 1\right] = T_s\\
S^{02} &= \frac{-e\rho}{1+b^2}\\
S^{12} &= \frac{-b\rho}{1+b^2}\left(\frac{(1+b^2-e^2)\tanh^2\left(\frac{\sqrt{1+b^2-e^2}\,t}{l_s\sqrt{k(1+b^2)}}\right)}{1+b^2} - 1\right).
\end{aligned} \tag{2.11}$$

The above result reduces to the expressions in [8, 11] in the absence of an electromagnetic field. The off-diagonal elements of $T^{\mu\nu}$ can be recognized as the Poyntin tensor since we have orthogonal electric and magnetic field. In fact, the constant electromagnetic field background can be thought of being generated from an ideal liquid of fundamental strings, each stretching along $x^0, x^1$ and rigid flowing along $x^2$. Boosting to the rest frame of fundamental strings will make $b = 0$, diagonalizing $T^{\mu\nu}$. Taking $t \to \infty$, the late-time behavior is $R \to 0$ and:

$$\begin{aligned} T^{00} &= \rho \\ T^{11}_{\infty} &= \frac{-\rho e^2}{(1+b^2)^2} \\ T^{22}_{\infty} &= \frac{\rho e^2 b^2}{(1+b^2)^2} \\ T^{02}_{\infty} &= \frac{eb\rho}{1+b^2} \\ T^{33}_{\infty} &= 0 \\ S^{02}_{\infty} &= \frac{-e\rho}{1+b^2} \\ S^{12}_{\infty} &= \frac{-be^2\rho}{(1+b^2)^2}\,. \end{aligned} \tag{2.12}$$

It is easy to see that in (2.11)

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{rt} + T^{\mu\nu}_{\infty} \\ T^{\mu\nu}_{rt} &= \mathrm{diag}[\rho, T_s, T_s, T_s] \\ T_s &= \frac{\rho}{1+b^2}(1+b^2-e^2)\left[\tanh^2\left(\frac{\sqrt{1+b^2-e^2}\;t}{l_s\sqrt{k(1+b^2)}}\right)-1\right]. \end{aligned} \tag{2.13}$$

This is a well-known phenomenon: when turning on an electromagnetic field in the world-volume of an unstable Dbrane, the Dbrane decays away, leaving out the string fluid. So the energy-momentum tensor is the sum of two parts: one is for the decaying Dbrane as $T_{rt}$, which represents the asymptotic pressureless matter; and the second part $T_\infty$ for the string fluid which is independent of time.

Several remarks are in order:

1. This bounded solution exists when the conserved energy is not too large to make the Dbrane escape the attractive gravitational force. Actually, from (2.11), we notice that
$$H\dot{r}^2(1+b^2) = 1+b^2-e^2-\frac{\tau_p^2(1+b^2)^2}{H\rho^2}\,, \tag{2.14}$$
Dbrane approaches its largest distance $r$ from NS5-branes at the turning point $\dot{r}|_{r_{max}} = 0$. We want our solution bounded in the near throat region, that is $r_{max} \ll l_s\sqrt{k}$ or $H(r_{max}) \gg 1$. From (2.14), we obtain:
$$H(r_{\max}) = \frac{\tau_p^2}{\rho^2}\frac{(1+b^2)^2}{1+b^2-e^2} \gg 1$$

$$\Rightarrow \frac{\rho}{\tau_p} \ll \frac{1+b^2}{\sqrt{1+b^2-e^2}}\,. \tag{2.15}$$

On the other hand, the string coupling should also be small in order that our tree level analysis applies, which gives the bound

$$\exp(\Phi) = A\cosh\frac{\sqrt{1+b^2-e^2}t}{l_s\sqrt{k(1+b^2)}} \ll 1 \Rightarrow \frac{\rho}{\tau_p} \gg \frac{g_s(1+b^2)}{\sqrt{1+b^2-e^2}}\,. \tag{2.16}$$

So in the case

$$\frac{g_s(1+b^2)}{\sqrt{1+b^2-e^2}} \ll \frac{\rho}{\tau_p} \ll \frac{1+b^2}{\sqrt{1+b^2-e^2}}\,, \tag{2.17}$$

the near throat approximation applies, and there is a long period of time in which the tree level analysis is good.

2. For an observer on NS5-branes at asymptotic infinity, the velocity of D$p$-brane is

$$\begin{aligned}\mathcal{V} &= \frac{dR}{dt} = \frac{\sqrt{1+b^2-e^2}}{A\sqrt{1+b^2}}\frac{\tanh\frac{\sqrt{1+b^2-e^2}t}{l_s\sqrt{k(1+b^2)}}}{\cosh\frac{\sqrt{1+b^2-e^2}t}{l_s\sqrt{k(1+b^2)}}} \\ &\underset{t\to\infty}{\sim\sim} \frac{1}{\cosh\frac{\sqrt{1+b^2-e^2}t}{l_s\sqrt{k(1+b^2)}}}\,.\end{aligned} \tag{2.18}$$

From the NS5-branes point of view, the velocity of the Dbrane slows down to zero when falling to the NS5-branes and the Dbrane will get close to the NS5-branes in an infinite amount of time. This is the familiar redshift phenomenon in general relativity.

3. On the other hand, the process seen by the observer on the D$p$-brane is quite different. Due to the existence of the electromagnetic field strength, the open string dynamics should be described by the open string metric $\mathcal{G}_{\mu\nu}$[19]. The comoving time $\tau$ is related to $t$ by $d\tau = \sqrt{-\mathcal{G}_{00}}dt$. Without loss of generality , we turn off the magnetic field $b$ (as argued above, we can boost to the rest frame of fundamental strings so as to make b=0) and then

$$\frac{d\tau}{dt} = \sqrt{1-e^2-(1-e^2)\left(\tanh^2\frac{\sqrt{1-e^2}\,t}{l_s\sqrt{k}}\right)} = \sqrt{(1-e^2)\frac{1}{\cosh^2\frac{\sqrt{1-e^2}\,t}{l_s\sqrt{k}}}} \tag{2.19}$$

which lead to

$$\begin{aligned}\tan\frac{\tau}{2l_s\sqrt{k}} &= e^{\frac{\sqrt{1-e^2}\,t}{l_s\sqrt{k}}} \\ R &= \frac{\sqrt{k}l_s}{A}\sin\frac{\tau}{l_s\sqrt{k}}\,.\end{aligned} \tag{2.20}$$

In the above, $\tau$ runs from 0 to $\sqrt{k}l_s\pi$. Therefore, from D$p$-brane's point of view, it approaches the NS5-branes in a finite amount of time, its velocity does not decay (of course, in the very late time, our analysis is invalidated by the strong string coupling).

Actually, the Dbrane seems to feel no effect of electric field strength since in terms of comoving time $\tau$, the motion $R(\tau)$ is independent of $e$ except the amplitude of the oscillation.

4. As usual, there exists a critical value for the electric field $e^2 \to 1 + b^2$, in which case the Lorentz factor $\sqrt{(1+b^2-e^2)/(1+b^2)}$ vanishes. And when it approaches this limit, the Dbrane stays still, since there is a balance between the gravitational attraction and the electromagnetic repulsion. More precisely, a good description of the Dbrane should be a NCOS theory [19].

5. Similarly, one may discuss the case when one transverse direction of the NS5-branes get compactified on a $S^1$[9]. We label the compact direction by $y$ and other three directions by $\vec{z}$. In the near throat region, for the scalar fields $y$ and $\vec{z}$, the effective potential is $(1 - \frac{e^2}{1+b^2})h^{-1}$, where

$$h = \frac{k}{2r|\vec{z}|}\frac{\sinh(|\vec{z}|/r)}{\cosh(|\vec{z}|/r) - \cos(y/r)}. \tag{2.21}$$

When $y = \pi r$ and $|\vec{z}| = 0$, there is no force on the D$p$-brane. But this is an unstable configuration, the fluctuations turn out to be of

$$m_y^2 \sim -\left(1 - \frac{e^2}{1+b^2}\right)\frac{1}{k}, \qquad m_z^2 \sim \left(1 - \frac{e^2}{1+b^2}\right)\frac{1}{k}. \tag{2.22}$$

Therefore, one has a tachyonic mode around $y = \pi r$. This unstable D$p$-brane configuration reminds us of non-BPS branes. A remarkable fact is that when $e$ gets close to the critical value, due to the balance between the gravitational attraction and the electromagnetic repulsion, the tachyon mode tends to be massless, reminiscing of NCOS.

## 3. Boundary states

The conformal field theory in the background of (2.1) is described by the CHS superconformal system in the near throat limit. From (2.1) it is easy to see the topology near horizon looks like $R^{1,5} \times R^1 \times S^3$, and the conformal field theory factorizes accordingly. The three factors are:

- $\mathbb{R}^{1,5}$ : the free string theory targeted in six dimensional Minkowski space or the NS5 brane world volume.

- $\mathbb{R}^1$ : a linear dilaton CFT with the dilaton proportional to proper distance $\phi$ : $\Phi = -\frac{\phi}{\sqrt{k}l_s}$.

- $S^3$ : the level $k$ SU(2) supersymmetric WZW model.

The boundary states describing the Dbrane in the configuration without an electromagnetic field were given in [10]. The $\mathrm{SU}(2)_k$ WZW theory in general allows $k+1$ different boundary states corresponding to BPS D-branes [20, 21]:

$$
\begin{aligned}
|B,l\rangle &= \frac{1}{\sqrt{2}}\sum_{2j=0}^{k}\left(\frac{S_j^l}{\sqrt{S_j^0}}|j\rangle\rangle_{\mathrm{NSNS}}+\frac{S_j^{\prime l}}{\sqrt{S_j^{\prime 0}}}|j\rangle\rangle_{\mathrm{RR}}\right)\\
|j\rangle\rangle_{\mathrm{NS,R}} &= \sum_{\psi\in\mathcal{H}_{\mathrm{NS,R}}} e^{i\phi(j,\psi)}|j,\psi\rangle_L|j,\psi\rangle_R\,, \qquad (3.1)
\end{aligned}
$$

where the sum goes over all left-right symmetric states constructed from SU(2) prime $|j\rangle$.

The space-like world-volume of the D-brane is the standard one (we neglect the ghost part through out):

$$
|B_{\mathrm{space}}\rangle = N_p\prod_{i=1}^{i=5} e^{\sum_n \frac{1}{n}\alpha^i_{-n}\bar{\alpha}^i_{-n}}\,. \qquad (3.2)
$$

Here $N_p$ is a normalization factor proportional to $T_p$. The fermionic part is a little complicated and not essential. We refer the reader to [22].

The remaining two dimensions $(\phi,t)$ are related as the on-shell motion of D-brane (2.8) with vanishing $e$ and $b$. This is described by the supersymmetric "hairpin brane" [23, 10]:

$$
|B;P,Q\rangle^\sigma = \int_0^\infty dp\int_{-\infty}^{\infty} d\omega\,\Psi^\sigma_{P,Q}(p,q)|p,\omega\rangle\rangle^\sigma \qquad (3.3)
$$

where $\sigma$ represents R-R sector or NS-NS sector sector, $|p,\omega\rangle\rangle$ is the Ishibashi states associated to some irreducible massive characters, and the wave functions are (as in [10], we focus on the case $P=Q=0$)

$$
\begin{aligned}
\Psi^{(NS)}_{P=0,Q=0}(p,\omega) &= \frac{-i\sqrt{2}\mathcal{Q}e^{i\frac{2p}{\mathcal{Q}}ln\tilde{r}}\sinh(\frac{2\pi p}{\mathcal{Q}})}{2\cosh[\frac{\pi}{\mathcal{Q}}(p+\omega)]\cosh[\frac{\pi}{\mathcal{Q}}(p-\omega)]}\bullet\frac{\Gamma(-i\mathcal{Q}p)\Gamma(1-i\frac{2p}{\mathcal{Q}})}{\Gamma(\frac{1}{2}+i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(\frac{1}{2}-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})} \qquad (3.4)\\
\Psi^{(NS)}_{P=0,Q=0}(\phi,t) &= \frac{\sqrt{2}}{\pi\mathcal{Q}(2\cosh\frac{\mathcal{Q}t}{2})^{\frac{2}{\mathcal{Q}^2}+1}}\exp\left[-\frac{\phi'}{\mathcal{Q}}-\frac{e^{-\frac{\phi'}{\mathcal{Q}}}}{(2\cosh\frac{\mathcal{Q}t}{2})^{\frac{2}{\mathcal{Q}^2}}}\right],\ \phi'=\phi-\frac{2}{\mathcal{Q}}\ln\tilde{r} \qquad (3.5)\\
\Psi^{(R)}_{P=0,Q=0}(p,\omega) &= \frac{-i\sqrt{2}\mathcal{Q}e^{i\frac{2p}{\mathcal{Q}}ln\tilde{r}}\sinh(\frac{2\pi p}{\mathcal{Q}})}{\cosh\frac{2\pi p}{\mathcal{Q}}-\cosh\frac{2\pi\omega}{\mathcal{Q}}}\bullet\frac{\Gamma(-i\mathcal{Q}p)\Gamma(1-i\frac{2p}{\mathcal{Q}})}{\Gamma(1-i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})\Gamma(i\frac{\omega}{\mathcal{Q}}-i\frac{p}{\mathcal{Q}})} \qquad (3.6)\\
\Psi^{(R)}_{P=0,Q=0}(\phi,t) &= e^{\mathcal{Q}t}2\frac{\sqrt{2}}{\pi\mathcal{Q}(2\cosh\frac{\mathcal{Q}t}{2})^{\frac{2}{\mathcal{Q}^2}+1}}\exp\left[-\frac{\phi'}{\mathcal{Q}}-\frac{e^{-\frac{\phi'}{\mathcal{Q}}}}{(2\cosh\frac{\mathcal{Q}t}{2})^{\frac{2}{\mathcal{Q}^2}}}\right],\ \phi'=\phi-\frac{2}{\mathcal{Q}}\ln\tilde{r}\,, \qquad (3.7)
\end{aligned}
$$

where $\mathcal{Q}=\frac{2}{l_s\sqrt{k}}$, $\tilde{r}=\frac{2}{A}$. Note that the NS sector wave function and R sector wave function are related by the spectrum flow:

$$
\begin{aligned}
\Psi^{(NS)}(p,\omega) &= \Psi^{(R)}\left(p,\omega-i\frac{\mathcal{Q}}{2}\right) \qquad (3.8)\\
\Psi^{(NS)}_{P=0,Q=0}(\phi,t) &= e^{-\mathcal{Q}t}2\Psi^{(R)}_{P=0,Q=0}(\phi,t)\,. \qquad (3.9)
\end{aligned}
$$

Now we apply the T-duality approach developed in [22, 18] to obtain the microscopic description of the Dbrane with the electromagnetic field turned on. For simplicity, we set $b = 0$ first and perform a chain of maps, which retains the solvability of the BCFT.

Step 1: In the absence of an electromagnetic field, compactify $x_1$ on a circle of radius $R_1$, and T-dual along $x_1$. The left and right moving parts of the world sheet scalar fields $X^a(z,\bar{z})$ are denoted by $X^a_L, X^a_R$. Under the T-duality

$$\begin{aligned} \begin{pmatrix} X^0_L \\ X^1_L \end{pmatrix} &\mapsto \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} X^0_L \\ X^1_L \end{pmatrix} \\ \begin{pmatrix} X^0_R \\ X^1_R \end{pmatrix} &\mapsto \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^0_R \\ X^1_R \end{pmatrix} \end{aligned} \tag{3.10}$$

In this process, a D$p$-brane turns into an array of D(p-1)-branes on a circle of radius $\tilde{R}_1 = 2\pi/R_1$. Take $\tilde{R}_1 \to \infty$ limit, isolating a localized D(p-1)-brane, the boundary state for $X^1$ is turned into a Dirichlet one:

$$|B, X^1\rangle = \exp\left(+\sum_{n=1}^{\infty} \frac{1}{n} \alpha^1_{-n} \bar{\alpha}^1_{-n}\right) \delta(\hat{x}^1)|0\rangle \tag{3.11}$$

Step 2: Boost the D(p-1)-brane along $x^1$ to a velocity $e$, the world sheet fields are then transformed as

$$\begin{aligned} \begin{pmatrix} X^0_L \\ X^1_L \end{pmatrix} &\mapsto \gamma \begin{pmatrix} 1 & e \\ e & 1 \end{pmatrix} \begin{pmatrix} X^0_L \\ X^1_L \end{pmatrix} \\ \begin{pmatrix} X^0_R \\ X^1_R \end{pmatrix} &\mapsto \gamma \begin{pmatrix} 1 & e \\ e & 1 \end{pmatrix} \begin{pmatrix} X^0_R \\ X^1_R \end{pmatrix}, \end{aligned} \tag{3.12}$$

where $\gamma = 1/\sqrt{1-e^2}$ is a Lorentz contraction factor. After this operation, the oscillators in the boundary state (3.11) are transformed in the same way. But because of the constraint of $\hat{x}^1$ in (3.11) the zero mode transforms as:

$$\begin{aligned} \hat{x}^0 &\mapsto \gamma^{-1} x^0 \\ \delta(\hat{x}^1) &\mapsto \gamma^{-1} \delta(\hat{x}^1 + e\hat{x}^0) \,. \end{aligned} \tag{3.13}$$

It is clear that the boundary state of the hairpin is easily transformed, since $\hat{x}^0$ is not mixed with $\hat{x}^1$, just Lorentz contracts.

Step 3: T-dual along $X^1$ again, and we return to a D$p$-brane. Under the T-duality:

$$\begin{aligned} \begin{pmatrix} X^0_L \\ X^1_L \end{pmatrix} &\mapsto \begin{pmatrix} +1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} X^0_L \\ X^1_L \end{pmatrix} \\ \begin{pmatrix} X^0_R \\ X^1_R \end{pmatrix} &\mapsto \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^0_R \\ X^1_R \end{pmatrix}. \end{aligned} \tag{3.14}$$

Moreover, the zero mode part of the boundary state transforms as:

$$\gamma^{-1}\delta(\hat{x}^1 + e\hat{x}^0) = \gamma^{-1} \sum_{n\in\mathcal{Z}} e^{i\frac{n}{\tilde{R}_1}(\hat{x}^1 + e\hat{x}^0)}|0\rangle \mapsto \gamma^{-1} \sum_{m\in\mathcal{Z}} e^{imR_1(\hat{x}^1_L - \hat{x}^1_R + e\hat{x}^0)}|0\rangle \,. \tag{3.15}$$

In the decompactification limit, $R_1 \to \infty$, only $m = 0$ term contributes, yielding a Born-Infeld factor to the boundary state: $\sqrt{1-e^2}|0\rangle$. After this operation, we come back to a D$p$-brane but with a constant electric field $F_{01} = e$ turned on. Also, the timelike direction is lorentz contracted as in (3.13).

When turning on both the electric and magnetic fields, we have:

$$|0\rangle \mapsto \tilde{\gamma}^{-1}\gamma^{-1}|0\rangle \tag{3.16}$$

$$t \mapsto \gamma^{-1}x^0 \tag{3.17}$$

$$\begin{pmatrix} \alpha^0_{-n} \\ \alpha^1_{-n} \\ \alpha^2_{-n} \end{pmatrix} \mapsto \Lambda^{-1}\Omega^{-1} \begin{pmatrix} \alpha^0_{-n} \\ \alpha^1_{-n} \\ \alpha^2_{-n} \end{pmatrix} \tag{3.18}$$

$$\begin{pmatrix} \bar{\alpha}^0_{-n} \\ \bar{\alpha}^1_{-n} \\ \bar{\alpha}^2_{-n} \end{pmatrix} \mapsto \Lambda\Omega \begin{pmatrix} \bar{\alpha}^0_{-n} \\ \bar{\alpha}^1_{-n} \\ \bar{\alpha}^2_{-n} \end{pmatrix} \tag{3.19}$$

$$\begin{pmatrix} \psi^0_{-n} \\ \psi^1_{-n} \\ \psi^2_{-n} \end{pmatrix} \mapsto \Lambda^{-1}\Omega^{-1} \begin{pmatrix} \psi^0_{-n} \\ \psi^1_{-n} \\ \psi^2_{-n} \end{pmatrix} \tag{3.20}$$

$$\begin{pmatrix} \bar{\psi}^0_{-n} \\ \bar{\psi}^1_{-n} \\ \bar{\psi}^2_{-n} \end{pmatrix} \mapsto \Lambda\Omega \begin{pmatrix} \bar{\psi}^0_{-n} \\ \bar{\psi}^1_{-n} \\ \bar{\psi}^2_{-n} \end{pmatrix} \tag{3.21}$$

where

$$\Lambda = \begin{pmatrix} \gamma & 0 & \gamma e' \\ 0 & 1 & 0 \\ \gamma e' & 0 & \gamma \end{pmatrix}, \qquad \Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \tilde{\gamma} & -\tilde{\gamma} b \\ 0 & \tilde{\gamma} b & \tilde{\gamma} \end{pmatrix}$$
$$\gamma = \frac{1}{\sqrt{1-e'^2}}, \qquad \tilde{\gamma} = \frac{1}{\sqrt{1+b^2}}, \qquad e' = \frac{e}{\sqrt{1+b^2}}. \tag{3.22}$$

It is not hard to see that the modified hairpin solution (2.8) is consistent with the transformation (3.17). Therefore, we propose here that when the electromagnetic field is turned on, the hairpin brane gets modified as in (2.8), and the boundary state can be obtained in a simple way: the Ishibashi states are modified only by the rescaling (3.17), and the boundary wave function $\Psi^{\sigma}_{P,Q}(p,\omega) \mapsto \Psi^{\sigma}_{P,Q}(p,\gamma\omega)$.[2] One subtle point is that we are short of a rigorous proof that the modified hairpin (2.8) is a marginal deformation. Nevertheless, the experience on the rolling tachyon tells us that the T-duality approach gives us the correct boundary states with the electromagnetic fields turning on. We believe that the modified hairpin is a truly marginal operator and our proposal is correct.

The same Fourier transformation procedure as in [10] leads to the coordinate space wave function which peaks at the trajectory (2.8). This is a consistent check of our proposal.

The boundary states for other space-like directions without the electromagnetic field remain the same, while the boundary states for $X^{1,2}, \psi^{1,2}$ now are written as:

$$|B, X^i, i=1,2\rangle = N_p e^{\sum_n \frac{1}{n}\alpha'^i_{-n}\bar{\alpha}'^i_{-n}}, \qquad \alpha'^i_{-n} = [(\Omega\Lambda)^{-1}]^i_j \alpha^j, \qquad \bar{\alpha}'^i_{-n} = (\Lambda\Omega)^i_j \bar{\alpha}^j, \tag{3.23}$$

[2] Here p is the momentum in the $R_\phi$ direction as in [11].

with the fermionic part transforming in the same way. There are factors $\gamma, \tilde{\gamma}$ in the oscillator part, we can choose a gauge in which the contribution to the total energy and particle number of closed string radiation comes only from timelike zero modes [5], so that only the vacuum rescaling in (3.16) is relevant.

Now we are ready to calculate the closed string emission rate from the above boundary states.

## 4. Closed string radiation: without winding strings

Note first that the amplitude for the emission of a closed string mode $V$ is

$$\mathcal{A} \sim \frac{\langle V|B\rangle}{\sqrt{E}}\,, \tag{4.1}$$

where $\langle V|B\rangle$ is the one-point function of a closed string vertex operator. One should also keep in mind that only the left-right symmetric closed string states are contained in the spectrum of a Dbrane. In order to calculate the above one-point function, one needs a full description of the boundary state. The boundary state factorizes into three independent pieces:

- The hairpin part is already given above, and its contribution is determined by the wave function;
- The boundary state of the spacelike directions without the electromagnetic field is the same as the standard one, while the one of the directions involving the electromagnetic field get changed to (3.23);
- As shown in (3.1), the SU(2)$WZW$ part contributes a phase factor as well as an overall constant factor $S^i_j S'^i_j$. Without loss of generality, we take $l = 0$ in (3.1), that is a Dbrane localized in $S^3$ (the group manifold of SU(2)).

Now the cross-section is

$$\begin{aligned}
|\langle V(j,p,\omega)|B\rangle|^2 &\simeq \gamma^{-2}\tilde{\gamma}^{-2}\frac{\pi}{k}\frac{\sinh(\pi\sqrt{k}p)}{(\cosh\pi\sqrt{k}\gamma\omega + \cosh\pi\sqrt{k}p)\sinh\frac{\pi p}{\sqrt{k}}}S^j_0 \\
\omega &= \sqrt{p^2 + k_\perp^2 + 4n + \frac{j(j+1)}{k}}\,,
\end{aligned} \tag{4.2}$$

where $p$ is the momentum in the R (linear dilaton) direction, $\vec{k}_\perp$ is the momentum parallel to the NS5-branes and transverse to the Dbrane, $n$ is the total oscillator level and $j$ is the SU(2) spin of the primary state we consider here.

We are interested in the total average number and energy of particles emitted by the Dbrane, which are given by

$$\frac{\mathcal{N}}{V} = \sum_{\text{states}} \frac{1}{2\omega}|\langle V(j,p,\omega)|B\rangle|^2 \tag{4.3}$$

$$\frac{\mathcal{E}}{V} = \sum_{\text{states}} |\langle V(j,p,\omega)|B\rangle|^2\,. \tag{4.4}$$



JHEP12(2004)057

From the form of the above cross-section, the possible divergence may come from the large $p$ region. Thus $\frac{j(j+1)}{k}$ in $\omega$ can be neglected in this region. Recall that the density of left-right symmetric closed string states for large level $n$ is $d_n \sim n^{-\frac{q+1}{2}} e^{\pi\sqrt{4n}\sqrt{\frac{2k-1}{k}}}$, where $2q = 6$ is the number of non-compact spatial directions. So another possible divergence could be from the exponential growth of the states with large level. Set $l^2 = 4n$, and change the summation over $n$ into the integral over $l$, the average number of total emitted particle could be approximated by:

$$\frac{\mathcal{N}}{V} \sim \gamma^{-2}\tilde{\gamma}^{-2} \int d\theta d\phi dr (\cos\theta)^{d-1} (\sin\theta)^{1-q} (\sin\phi)^{-q} r^{d-q} \times$$
$$\times \exp\left[-\pi r\left(\gamma - \sin\theta\left(\left(1-\frac{1}{k}\right)\cos\phi + \sqrt{\frac{2k-1}{k^2}}\sin\phi\right)\right)\right] \quad (4.5)$$

after introducing a set of spherical coordinates

$$\begin{aligned} r &= \sqrt{k}\omega, \\ p\sqrt{k} &= r\sin\theta\cos\phi, \\ l\sqrt{k} &= r\sin\theta\sin\phi, \\ k_\perp\sqrt{k} &= r\cos\theta\,. \end{aligned} \quad (4.6)$$

Here $d$ is the number of flat space-like directions transverse to the Dbrane in NS5-branes.[3] The expression in the exponential in the integrand could be written as

$$-\pi r\left(\gamma - \sin\theta\left(\left(1-\frac{1}{k}\right)\cos\phi + \sqrt{\frac{2k-1}{k^2}}\sin\phi\right)\right) = -\pi r(\gamma - \sin\theta\cos(\phi-\chi))\,, \quad (4.7)$$

where

$$\cos\chi = 1 - \frac{1}{k}\,, \qquad \sin\chi = \sqrt{\frac{2k-1}{k^2}}\,. \quad (4.8)$$

Since $\gamma > 1$ when $e \neq 0$, the integral is always finite and the steepest descent approximation gives:

$$\begin{aligned} \frac{\mathcal{N}}{V} &\sim \gamma^{-2}\tilde{\gamma}^{-2} \int dr \int dx r^{d-q} x^{d-1} e^{-\pi r(\gamma - 1 + \frac{1}{2}x^2)} \\ &\sim \gamma^{-2}\tilde{\gamma}^{-2} \int dr e^{-\pi r(\gamma-1)} r^{d/2-q} \\ &\sim \gamma^{-2}\tilde{\gamma}^{-2} (\gamma-1)^{\frac{4-d}{2}}\,, \end{aligned} \quad (4.9)$$

Similarly, one can obtain the total emitted energy

$$\frac{\mathcal{E}}{V} \sim \gamma^{-2}\tilde{\gamma}^{-2} \int dr e^{-\pi r(\gamma-1)} r^{d/2-q+1} \sim \gamma^{-2}\tilde{\gamma}^{-2}(\gamma-1)^{\frac{2-d}{2}}. \quad (4.10)$$

It is obvious that when

[3] Here for D$p$-brane, d=5-p.

1. $e \neq 0$ (especially this is independent of the magnetic field), the above formula is finite thus indicates the validity of our tree level analysis.

2. $e \to 0$, that is $\gamma \to 1$. For the D$p$-brane, $d = 5 - p$, when $p > 3$ the total emitted energy is finite but for $p \leq 3$ it is infinite, implying the breakdown of the tree level treatment.

3. $e$ tends to the critical value $e = 1 + b^2$, $\gamma \to \infty$, the emitted energy $\mathcal{E} \sim \tilde{\gamma}^{-2}\gamma^{\frac{-d}{2}} \to 0$. This means that D$p$-brane decouples from the closed string.

## 5. Closed string radiation: with winding strings

The above result shows that the total energy of the closed string radiation is finite with the presence of nonvanishing electric field. This is similar to what happens in the rolling tachyon case [14]. However, the authors of [15] found that in the rolling tachyon case, taking the winding closed string into account the radiation is UV divergent when $p \leq 3$, and as a result the emitted winding closed string will dominate the tachyon matter.

For simplicity, we take $b = 0$ (so $\tilde{\gamma} = 1, \gamma = \frac{1}{\sqrt{1-e^2}}$). Let us compactify $x^1$ direction on a circle of radius R, and wrap the Dbrane on this circle. The boundary state for zero mode is

$$\begin{aligned} |B, m\rangle^\sigma &= \sqrt{1-e^2}\sum_m \int_0^\infty dp \int_{-\infty}^\infty d\omega \Psi^\sigma(p,\omega) \cdot \\ & |p, \omega_s = p_L^0 = p_R^0 = \sqrt{1-e^2}\omega + eRm, p_L^1 = -p_R^1 = Rm\rangle\rangle^\sigma \end{aligned} \tag{5.1}$$

where $m$ is the winding number. Now for a left-right symmetric closed string with winding number $m$ and energy $\omega$, the cross section is

$$\begin{aligned} |\langle V(j,p,m,\omega_s)|B\rangle|^2 &\simeq \gamma^{-2}\frac{\pi^2}{k}\frac{\sinh(\pi\sqrt{k}p)}{(\cosh \pi\sqrt{k}\gamma(\omega - eRm) + \cosh \pi\sqrt{k}p)\sinh\frac{\pi p}{\sqrt{k}}}S_0^j \\ \omega_s &= \sqrt{p^2 + k_\perp^2 + 4n + m^2R^2 + \frac{j(j+1)}{k}}\,, \end{aligned} \tag{5.2}$$

and the average number and energy of total emitted particles are

$$\frac{\mathcal{N}}{V} = \sum_{\text{states}} \frac{1}{2\omega_s}|\langle V(j,p,m,\omega_s)|B\rangle|^2 \tag{5.3}$$

$$\frac{\mathcal{E}}{V} = \sum_{\text{states}} |\langle V(j,p,m,\omega_s)|B\rangle|^2\,. \tag{5.4}$$

Note that now the summation over states include not only all states of different levels but also of different windings.

As in the above section, we are interested in the high energy region so we can neglect the term $j(j+1)/k$ in $\omega_s$. The total emitted energy is

$$\frac{\mathcal{E}}{V} \sim R(1-e^2)\sum_m \int dpdld^dk_\perp l^{-q}e^{l\pi\sqrt{\frac{2k-1}{k}}} \cdot e^{-\pi\sqrt{k}\gamma(\sqrt{p^2+k_\perp^2+l^2+m^2R^2+\frac{j(j+1)}{k}} - eRm) + \pi\sqrt{k}p - \frac{p\pi}{\sqrt{k}}} \tag{5.5}$$

where $l^2 = 4n$. In the large $n$ limit, we can replace the sum over $m$ to the integral. Introduce the spherical coordinates

$$\begin{aligned} r &= \sqrt{k}\sqrt{(mR)^2 + p^2 + l^2 + k_\perp^2} \\ mR\sqrt{k} &= r\cos\theta \\ k_\perp\sqrt{k} &= r\sin\theta\cos\varphi \\ l\sqrt{k} &= r\sin\theta\sin\varphi\cos\phi \\ p\sqrt{k} &= r\sin\theta\sin\varphi\sin\phi\,, \end{aligned} \tag{5.6}$$

the integral becomes

$$\begin{aligned} \frac{\mathcal{E}}{V} \sim \int dr d\theta d\varphi d\phi r^{d-q+2}(\sin\theta)^{d+1-q}(\sin\varphi)^{1-q}(\cos\varphi)^{d-1}(\cos\phi)^{-1} \times \\ \times \exp(-\pi r(\gamma(1-e\cos\theta) - \sin\theta\sin\varphi\sin(\phi-\chi))) \end{aligned} \tag{5.7}$$

where $\chi$ is defined in (4.8).

The factor in the exponential can be written as

$$-\pi r\gamma(1 - \cos\delta\cos\theta - \sin\delta\sin\theta\sin\varphi\sin(\phi-\chi)), \tag{5.8}$$

where

$$\cos\delta = e\,, \qquad \sin\delta = \sqrt{1-e^2}\,. \tag{5.9}$$

Therefore, we see that the integral has power-like divergence in $r$ when $\phi \sim \chi + \frac{\pi}{2}$, $\varphi \sim \frac{\pi}{2}$ and $\theta \sim \delta$. This contrasts with the unwinding case, where $\theta = \pi/2$ and the above integral is convergent. By using the steepest descent method, we find

$$\frac{\mathcal{E}}{V} \sim \int dr r^{\frac{d}{2}-q+2}\,, \tag{5.10}$$

which is always divergent. Similarly, one can calculate the average number of total emitted particles

$$\frac{\mathcal{N}}{V} \sim \int dr r^{\frac{d}{2}-q+1}\,. \tag{5.11}$$

Therefore, taking into account the wound strings, the total energy of emitted string is divergent. This suggests that the tachyon matter consists mainly of the highly wound strings.

There are two interesting limits in this case. When $e = 0$, namely turning off all the electromagnetic field, the dominant contribution in the exponential is from $\theta \simeq \delta = \pi/2$ and the integration by steepest descent method tells us that the total emitted energy is still divergent. On the other hand, when $e$ reach the critical value, $\gamma \to \infty$ and it is not hard to see that the total emitted energy vanish.

## 6. Conclusion

In this paper, we investigated the dynamics of a D$p$-brane probe near a stack of NS5-branes with constant electromagnetic field turned on. The result is similar to but differs in detail from the rolling tachyon case: Dbrane falls onto the NS5-branes with a speed approaching a maximum velocity less than the speed of light (as seen by the near throat observer), radiating closed string modes, leaving out the string fluid on the NS5-branes. The whole process is slowed by a Lorentz factor $\gamma > 1$. The average number and energy of the emitted particle are both finite when electric field $e$ and magnetic field $b$ satisfy $0 < e < \sqrt{1+b^2}$. When compactify the Dbrane on a circle and take into account the winding string radiation, we find that the total radiation always diverges, implying that the winding string emission dominates the tachyon matter. We also study several properties of the system when $e$ approaches the critical value $\sqrt{1+b^2}$, a limit related to NCOS theory, and this deserves further study.

A more geometric system was constructed in [8] with one transverse direction compactified. Our DBI analysis could be easily generalized to that case, but the conformal field theory of the system turns to be quite complicated so we are not able to construct the boundary state of the Dbrane. We leave this problem as a future plan.

Another interesting question is to analyze the modified hairpin brane (2.8) in the context of the conformal field theory. It would be nice to prove it is truly marginal and construct its boundary state directly from BCFT.


## Acknowledgments

The work of M.L. and B.S. was supported by a grant of NFSC and a grant of Chinese Academy of Sciences, the work of B.C. was supported by a grant of Chinese Academy of Sciences.


## References


[1] A. Sen, *Rolling tachyon*, *J. High Energy Phys.* **04** (2002) 048 [hep-th/0203211].

[2] M. Gutperle and A. Strominger, *Spacelike branes*, *J. High Energy Phys.* **04** (2002) 018 [hep-th/0202210].

[3] B. Chen, M. Li and F.L. Lin, *Gravitational radiation of rolling tachyon*, *J. High Energy Phys.* **11** (2002) 050 [hep-th/0209222].

[4] T. Okuda and S. Sugimoto, *Coupling of rolling tachyon to closed strings*, *Nucl. Phys.* **B 647** (2002) 101 [hep-th/0208196].

[5] N. Lambert, H. Liu and J. Maldacena, *Closed strings from decaying D-branes*, hep-th/0303139.

[6] A. Sen, *Tachyon matter*, *J. High Energy Phys.* **07** (2002) 065 [hep-th/0203265]; *Field theory of tachyon matter*, *Mod. Phys. Lett.* **A 17** (2002) 1797 [hep-th/0204143].

[7] I. Bars, I. Kishimoto and Y. Matsuo, *Fermionic ghosts in moyal string field theory*, *J. High Energy Phys.* **07** (2003) 027 [hep-th/0304005];
V. Niarchos, *Notes on tachyon effective actions and veneziano amplitudes*, *Phys. Rev.* **D 69** (2004) 106009 [hep-th/0401066].

[8] D. Kutasov, *D-brane dynamics near NS5-branes*, hep-th/0405058.

[9] D. Kutasov, *A geometric interpretation of the open string tachyon*, hep-th/0408073.

[10] Y. Nakayama, Y. Sugawara and H. Takayanagi, *Boundary states for the rolling D-branes in NS5 background*, *J. High Energy Phys.* **07** (2004) 020 [hep-th/0406173].

[11] D.A. Sahakyan, *Comments on D-brane dynamics near NS5-branes*, *J. High Energy Phys.* **10** (2004) 008 [hep-th/0408070].

[12] D. Bak, S.J. Rey and H.U. Yee, *Exactly soluble dynamics of $(p, q)$ string near macroscopic fundamental strings*, *J. High Energy Phys.* **12** (2004) 008 [hep-th/0411099];
J. Kluson, *Non-BPS Dp-brane in the background of ns5-branes on transverse* $\mathbb{R}^3 \times S^1$, hep-th/0411014;
S. Thomas and J. Ward, *D-brane dynamics and NS5 rings*, hep-th/0411130;
A. Ghodsi and A.E. Mosaffa, *D-brane dynamics in RR deformation of NS5-branes background and tachyon cosmology*, hep-th/0408015;
K.L. Panigrahi, *D-brane dynamics in Dp-brane background*, *Phys. Lett.* **B 601** (2004) 64 [hep-th/0407134];
H. Yavartanoo, *Cosmological solution from D-brane motion in NS5-branes background*, hep-th/0407079.

[13] G.W. Gibbons, K. Hori and P. Yi, *String fluid from unstable D-branes*, *Nucl. Phys.* **B 596** (2001) 136 [hep-th/0009061];
G. Gibbons, K. Hashimoto and P. Yi, *Tachyon condensates, carrollian contraction of Lorentz group and fundamental strings*, *J. High Energy Phys.* **09** (2002) 061 [hep-th/0209034].

[14] K. Nagami, *Closed string emission from unstable D-brane with background electric field*, *J. High Energy Phys.* **01** (2004) 005 [hep-th/0309017].

[15] M. Gutperle and P. Yi, *Winding strings and decay of D-branes with flux*, hep-th/0409050.

[16] C.G. Callan Jr., J.A. Harvey and A. Strominger, *Supersymmetric string solitons*, hep-th/9112030.

[17] P. Mukhopadhyay and A. Sen, *Decay of unstable D-branes with electric field*, *J. High Energy Phys.* **11** (2002) 047 [hep-th/0208142].

[18] S.J. Rey and S. Sugimoto, *Rolling tachyon with electric and magnetic fields: T-duality approach*, *Phys. Rev.* **D 67** (2003) 086008 [hep-th/0301049].

[19] N. Seiberg, L. Susskind and N. Toumbas, *Strings in background electric field, space/time noncommutativity and a new noncritical string theory*, *J. High Energy Phys.* **06** (2000) 021 [hep-th/0005040];
R. Gopakumar, J.M. Maldacena, S. Minwalla and A. Strominger, *S-duality and noncommutative gauge theory*, *J. High Energy Phys.* **06** (2000) 036 [hep-th/0005048].

[20] S.A. Apikian and D.A. Sahakian, *Superconformal field theory with boundary: spin model*, *Mod. Phys. Lett.* **A 14** (1999) 211 [hep-th/9806107].

[21] R.I. Nepomechie, *Consistent superconformal boundary states*, *J. Phys.* **A 34** (2001) 6509 [hep-th/0102010].

[22] P. Di Vecchia and A. Liccardo, *D-branes in string theory, I*, *NATO Adv. Study Inst. Ser. C. Math. Phys. Sci.* **556** (2000) 1–59 [hep-th/9912161].

[23] S.L. Lukyanov, E.S. Vitchev and A.B. Zamolodchikov, *Integrable model of boundary interaction: the paperclip*, *Nucl. Phys.* **B 683** (2004) 423 [hep-th/0312168].